Chapter 6

Current and Resistance

Current and Resistance

6.1 Electric Current

Electric currents are flows of electric charge. Suppose a collection of charges is moving perpendicular to a surface of area *A*, as shown in Figure 6.1.1.

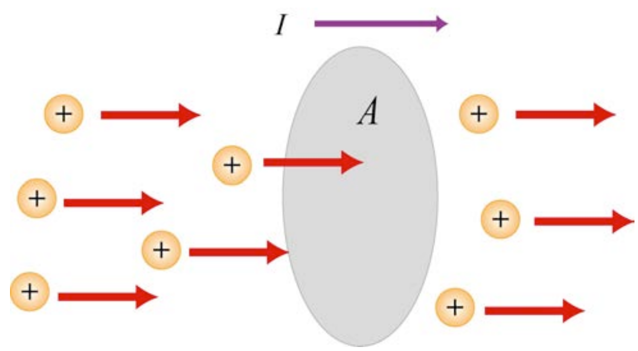

Figure 6.1.1 Charges moving through a cross section.

The electric current is defined to be the rate at which charges flow across any cross-sectional area. If an amount of charge ΔQ passes through a surface in a time interval Δt, then the average current I_{avg} is given by

$$I_{\text{avg}} = \frac{\Delta Q}{\Delta t}. \tag{6.1.1}$$

The SI unit of current is the ampere [A], with 1 A = 1 coulomb/sec. Common currents range from mega-amperes in lightning to nano-amperes in your nerves. In the limit $\Delta t \to 0$, the instantaneous current *I* may be defined as

$$I = \frac{dQ}{dt}. \tag{6.1.2}$$

Because flow has a direction, we have implicitly introduced a convention that the direction of current corresponds to the direction in which positive charges are flowing. The flowing charges inside wires are negatively charged electrons that move in the opposite direction of the current. Electric currents flow in conductors: solids (metals, semiconductors), liquids (electrolytes, ionized) and gases (ionized), but the flow is impeded in non-conductors or insulators.

6.1.1 Current Density

To relate current, a macroscopic quantity, to the microscopic motion of the charges, let's examine a conductor of cross-sectional area *A*, as shown in Figure 6.1.2.

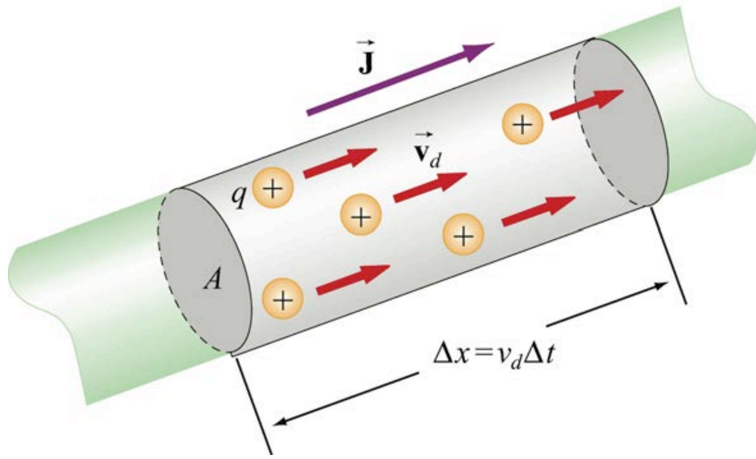

Figure 6.1.2 A microscopic picture of current flowing in a conductor.

Let the total current through a surface be written as

$$I = \iint \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}} \,. \tag{6.1.3}$$

where $\vec{\mathbf{J}}$ is the current density (the SI units of current density are $[\mathrm{A/m^2}]$). If q is the charge of each carrier, and n is the number of charge carriers per unit volume, the total amount of charge in this section is then $\Delta Q = q(nA\,\Delta x)$. Suppose that the charge carriers move with an average speed v_d; then the displacement in a time interval Δt will be $\Delta x = v_d \Delta t$, which implies

$$I_{\text{avg}} = \frac{\Delta Q}{\Delta t} = nqv_d A \,. \tag{6.1.4}$$

The average speed v_d at which the charge carriers are moving is known as the *drift speed*. Actually an electron inside the conductor does not travel in a straight line; instead, its path is rather erratic, as shown in Figure 6.1.3.

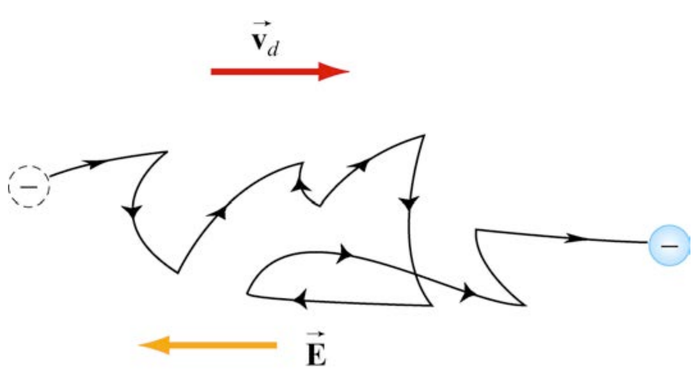

Figure 6.1.3 Motion of an electron in a conductor.

From the above equations, the current density $\vec{\mathbf{J}}$ can be written as

$$\vec{\mathbf{J}} = nq\vec{\mathbf{v}}_d . \tag{6.1.5}$$

Thus, we see that $\vec{\mathbf{J}}$ and $\vec{\mathbf{v}}_d$ point in the same direction for positive charge carriers, in opposite directions for negative charge carriers.

To find the drift velocity of the electrons, we first note that an electron in the conductor experiences an electric force $\vec{\mathbf{F}}_e = -e\vec{\mathbf{E}}$ that gives an acceleration

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}_e}{m_e} = -\frac{e\vec{\mathbf{E}}}{m_e} . \tag{6.1.6}$$

Denote the velocity of a given electron immediate after a collision by $\vec{\mathbf{v}}_i$. The velocity of the electron immediately before the next collision is then given by

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}\,t = \vec{\mathbf{v}}_i - \frac{e\vec{\mathbf{E}}}{m_e}t \tag{6.1.7}$$

where t is the time traveled. The average of $\vec{\mathbf{v}}_f$ over all time intervals is

$$\left\langle \vec{\mathbf{v}}_f \right\rangle = \left\langle \vec{\mathbf{v}}_i \right\rangle - \frac{e\vec{\mathbf{E}}}{m_e}\left\langle t \right\rangle \tag{6.1.8}$$

which is equal to the drift velocity $\vec{\mathbf{v}}_d$. Because in the absence of electric field, the velocity of the electron is completely random, it follows that $\left\langle \vec{\mathbf{v}}_i \right\rangle = 0$. If $\tau = \left\langle t \right\rangle$ is the average characteristic time between successive collisions (the *mean free time*), we have

$$\vec{\mathbf{v}}_d = \left\langle \vec{\mathbf{v}}_f \right\rangle = -\frac{e\vec{\mathbf{E}}}{m_e}\tau . \tag{6.1.9}$$

The current density in Eq. (6.1.5) becomes

$$\vec{\mathbf{J}} = -ne\vec{\mathbf{v}}_d = -ne\left(-\frac{e\vec{\mathbf{E}}}{m_e}\tau \right) = \frac{ne^2\tau}{m_e}\vec{\mathbf{E}} . \tag{6.1.10}$$

Note that $\vec{\mathbf{J}}$ and $\vec{\mathbf{E}}$ will be in the same direction for either negative or positive charge carriers.

6.2 Ohm's Law

In many materials, the current density is linearly dependent on the external electric field $\vec{\mathbf{E}}$,

$$\vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}, \tag{6.2.1}$$

where σ is called the *conductivity* of the material. The above equation is known as the (microscopic) Ohm's law. A material that obeys this relation is said to be ohmic; otherwise, the material is non-ohmic.

Comparing Eq. (6.2.1) with Eq. (6.1.10), we see that the conductivity can be expressed as

$$\sigma = \frac{ne^2\tau}{m_e}. \tag{6.2.2}$$

To obtain a more useful form of Ohm's law for practical applications, consider a segment of straight wire of length *l* and cross-sectional area *A*, as shown in Figure 6.2.1.

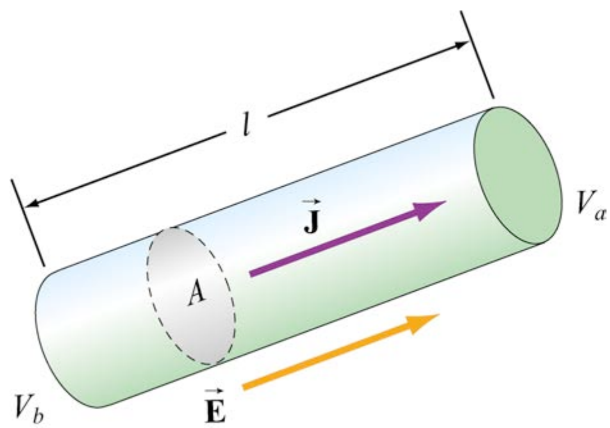

Figure 6.2.1 A uniform conductor of length *l* and potential difference $\Delta V = V_b - V_a$.

Suppose a potential difference $\Delta V = V_b - V_a$ is applied between the ends of the wire, creating an electric field $\vec{\mathbf{E}}$ and a current *I*. Assuming $\vec{\mathbf{E}}$ to be uniform, we then have

$$\Delta V = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = El\,. \tag{6.2.3}$$

The magnitude of the current density can then be written as

$$J = \sigma E = \sigma\left(\frac{\Delta V}{l}\right). \tag{6.2.4}$$

With $J = I/A$, the potential difference becomes

$$\Delta V = \frac{l}{\sigma} J = \left(\frac{l}{\sigma A} \right) I = RI, \tag{6.2.5}$$

where the *resistance* is given by

$$R = \frac{\Delta V}{I} = \frac{l}{\sigma A}. \tag{6.2.6}$$

The equation

$$\Delta V = IR \tag{6.2.7}$$

is the "macroscopic" version of the Ohm's law. The SI unit of R is the ohm $[\Omega]$, (Greek letter Omega), where

$$1\,\Omega \equiv \frac{1\,\text{V}}{1\,\text{A}}. \tag{6.2.8}$$

Once again, a material that obeys the above relation is ohmic, and non-ohmic if the relation is not obeyed. Most metals, with good conductivity and low resistivity, are ohmic. We shall focus mainly on ohmic materials. I ΔV

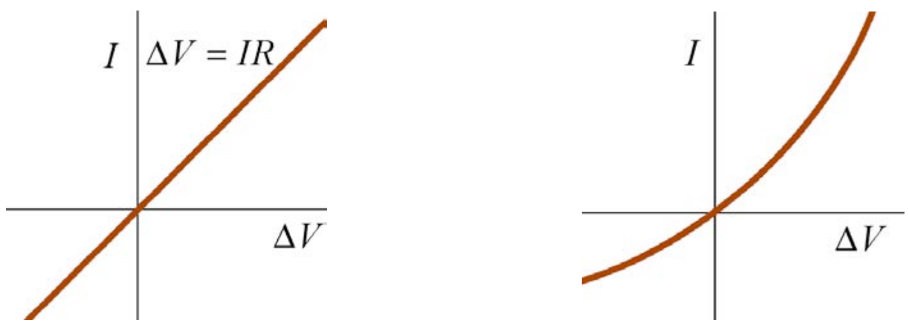

Figure 6.2.2 Ohmic vs. Non-ohmic behavior.

The resistivity ρ of a material is defined as the reciprocal of conductivity,

$$\rho = \frac{1}{\sigma} = \frac{m_e}{ne^2\tau}. \tag{6.2.9}$$

From the above equations, we see that ρ can be related to the resistance R of an object by

$$\rho = \frac{E}{J} = \frac{\Delta V / l}{I / A} = \frac{RA}{l}$$

or

$$R = \frac{\rho l}{A}. \tag{6.2.10}$$

The resistivity of a material actually varies with temperature T. For metals, the variation is linear over a large range of T:

$$\rho = \rho_0 \left[1 + \alpha (T - T_0) \right], \tag{6.2.11}$$

where α is the *temperature coefficient of resistivity*. Typical values of ρ, σ and α (at $20\,^\circ\mathrm{C}$) for different types of materials are given in the Table below.

Material	Resistivity ρ $(\Omega\cdot\mathrm{m})$	Conductivity σ $(\Omega\cdot\mathrm{m})^{-1}$	Temperature Coefficient α $(^\circ\mathrm{C})^{-1}$
Elements Silver	1.59×10^{-8}	6.29×10^{7}	0.0038
Copper	1.72×10^{-8}	5.81×10^{7}	0.0039
Aluminum	2.82×10^{-8}	3.55×10^{7}	0.0039
Tungsten	5.6×10^{-8}	1.8×10^{7}	0.0045
Iron	10.0×10^{-8}	1.0×10^{7}	0.0050
Platinum	10.6×10^{-8}	1.0×10^{7}	0.0039
Alloys Brass	7×10^{-8}	1.4×10^{7}	0.002
Manganin	44×10^{-8}	0.23×10^{7}	1.0×10^{-5}
Nichrome	100×10^{-8}	0.1×10^{7}	0.0004
Semiconductors Carbon (graphite)	3.5×10^{-5}	2.9×10^{4}	−0.0005
Germanium (pure)	0.46	2.2	−0.048
Silicon (pure)	640	1.6×10^{-3}	−0.075
Insulators Glass	$10^{10}-10^{14}$	$10^{-14}-10^{-10}$	
Sulfur	10^{15}	10^{-15}	
Quartz (fused)	75×10^{16}	1.33×10^{-18}	

6.3 Summary

- The **electric current** I is defined as:

$$I = \frac{dQ}{dt}.$$

- The **average current** I_{avg} in a conductor is

$$I_{\text{avg}} = nqv_d A$$

where n is the number density of the charge carriers, q is the charge each carrier has, v_d is the **drift speed**, and A is the cross-sectional area.

- The **current density** $\vec{\mathbf{J}}$ through the cross sectional area of the wire is

$$\vec{\mathbf{J}} = nq\vec{\mathbf{v}}_d.$$

- Microscopic Ohm's law: the current density is proportional to the electric field, and the constant of proportionality is called **conductivity** σ:

$$\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}.$$

- The reciprocal of conductivity σ is called **resistivity** ρ:

$$\rho = \frac{1}{\sigma}.$$

- Macroscopic Ohm's law: The **resistance** R of a conductor is the ratio of the potential difference ΔV between the two ends of the conductor and the current I:

$$R = \frac{\Delta V}{I}.$$

- Resistance is related to resistivity by

$$R = \frac{\rho l}{A}$$

where l is the length and A is the cross-sectional area of the conductor.

- The **drift velocity** $\vec{\mathbf{v}}_d$ of an electron in the conductor is

$$\vec{\mathbf{v}}_d = -\frac{e\vec{\mathbf{E}}}{m_e}\tau$$

where m_e is the mass of an electron, and τ is the average time between successive collisions.

- The resistivity of a metal is related to τ by

$$\rho = \frac{1}{\sigma} = \frac{m_e}{ne^2\tau}.$$

- The temperature variation of resistivity of a conductor is

$$\rho = \rho_0\left[1 + \alpha\left(T - T_0\right)\right]$$

where α is the **temperature coefficient of resistivity**.

- **Power**, or rate at which energy is delivered to the resistor is

$$P = I\Delta V = I^2 R = \frac{\left(\Delta V\right)^2}{R}.$$

6.4 Solved Problems

6.4.1 Resistivity of a Cable

A 3000-km long cable consists of seven copper wires, each of diameter 0.73 mm, bundled together and surrounded by an insulating sheath. Calculate the resistance of the cable. Use $3\times 10^{-6}\,\Omega\cdot\text{cm}$ for the resistivity of the copper.

Solution: The resistance R of a conductor is related to the resistivity ρ by $R = \rho l / A$, where l and A are the length of the conductor and the cross-sectional area, respectively. The cable consists of $N = 7$ copper wires, and so the total cross sectional area is

$$A = N\pi r^2 = N\frac{\pi d^2}{4} = 7\frac{\pi(0.073\text{cm})^2}{4}.$$

The resistance then becomes

$$R = \frac{\rho l}{A} = \frac{\left(3\times10^{-6}\,\Omega\cdot\text{cm}\right)\left(3\times10^{8}\text{cm}\right)}{7\pi\left(0.073\text{cm}\right)^{2}/4} = 3.1\times10^{4}\,\Omega\,.$$

6.4.2 Charge at a Junction

Show that the total amount of charge at the junction of the two materials in Figure 6.4.1 is $\varepsilon_0 I(\sigma_2^{-1} - \sigma_1^{-1})$, where I is the current flowing through the junction, and σ_1 and σ_2 are the conductivities for the two materials.

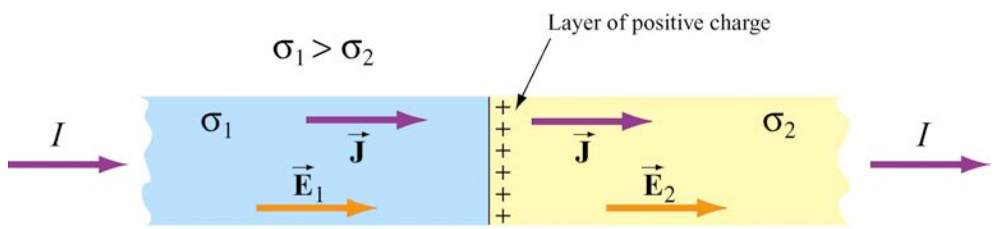

Figure 6.4.1 Charge at a junction.

Solution: In a steady state of current flow, the normal component of the current density $\vec{\mathbf{J}}$ must be the same on both sides of the junction. Since $J = \sigma E$, we have $\sigma_1 E_1 = \sigma_2 E_2$ or

$$E_2 = \left(\frac{\sigma_1}{\sigma_2}\right) E_1\,.$$

Let the charge on the interface boundary be q_{b}, we have, from the Gauss's law:

$$\oint\!\!\oint_S \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \left(E_2 - E_1\right)A = \frac{q_{\text{b}}}{\varepsilon_0}\,.$$

Thus

$$E_2 - E_1 = \frac{q_{\text{b}}}{A\varepsilon_0}\,.$$

Substituting the expression for E_2 from above yields

$$q_{\text{b}} = \varepsilon_0 A E_1\left(\frac{\sigma_1}{\sigma_2} - 1\right) = \varepsilon_0 A \sigma_1 E_1\left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right).$$

The current is $I = JA = (\sigma_1 E_1) A$, therefore the amount of charge on the interface boundary is

$$q_b = \varepsilon_0 I \left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1} \right).$$

6.4.3 Drift Velocity

The resistivity of seawater is about 25 $\Omega \cdot \text{cm}$. The charge carriers are chiefly Na^+ and Cl^- ions, and of each there are about $3 \times 10^{20} / \text{cm}^3$. If we fill a plastic tube 2 meters long with seawater and connect a 12-volt battery to the electrodes at each end, what is the resulting average drift velocity of the ions, in cm/s?

Solution:

The current in a conductor of cross sectional area *A* is related to the drift speed v_d of the charge carriers by

$$I = enAv_d,$$

where *n* is the number of charges per unit volume. We can then rewrite the Ohm's law as

$$V = IR = (neAv_d)\left(\frac{\rho l}{A} \right) = nev_d \rho l.$$

The drift velocity is then

$$v_d = \frac{V}{ne\rho l}.$$

Substituting the values, we have

$$\begin{aligned} v_d &= \frac{12\text{V}}{(6 \times 10^{20} \text{cm}^{-3})(1.6 \times 10^{-19} \text{C})(25\,\Omega \cdot \text{cm})(200 \text{ cm})} \\ &= 2.5 \times 10^{-5} \frac{\text{V} \cdot \text{cm}}{\text{C} \cdot \Omega} = 2.5 \times 10^{-5} \frac{\text{cm}}{\text{s}}. \end{aligned}$$

In converting the units we have used

$$\frac{\text{V}}{\Omega \cdot \text{C}} = \left(\frac{\text{V}}{\Omega} \right) \frac{1}{\text{C}} = \frac{\text{A}}{\text{C}} = \text{s}^{-1}.$$

6.4.4 Resistance of a Truncated Cone

Consider a material of resistivity ρ in a shape of a truncated cone of altitude h, and radii a and b, for the right and the left ends, respectively, as shown in the Figure 6.4.2. Assuming that the current is distributed uniformly throughout the cross-section of the cone, what is the resistance between the two ends?

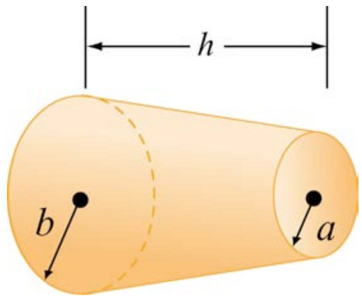

Figure 6.4.2 A truncated Cone.

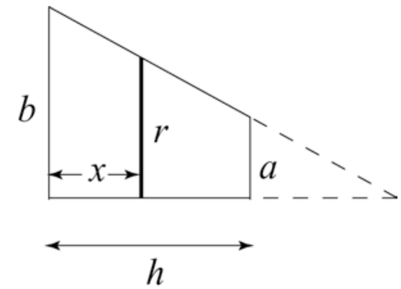

Figure 6.4.3

Solution: Consider a thin disk of radius r at a distance x from the left end. From the geometry illustrated in Figure 6.4.3, we have

$$\frac{b-r}{x}=\frac{b-a}{h}.$$

We can solve for the radius of the disk

$$r=(a-b)\frac{x}{h}+b.$$

The resistance R is related to resistivity ρ by $R=\rho l/A$, where l is the length of the conductor and A is the cross section. The contribution to the resistance from the disk having a thickness dy is

$$dR=\frac{\rho\,dx}{\pi r^2}=\frac{\rho\,dx}{\pi[b+(a-b)x/h]^2}.$$

Straightforward integration then yields

$$R=\int_0^h \frac{\rho\,dx}{\pi[b+(a-b)x/h]^2}=\frac{\rho h}{\pi ab},$$

where we have used

$$\int\frac{du}{(\alpha u+\beta)^2}=-\frac{1}{\alpha(\alpha u+\beta)}.$$

Note that if $b=a$, then area is $A=\pi a^2$, and set $h=l$, Eq.(6.2.10) is reproduced.

6.4.5 Resistance of a Hollow Cylinder

Consider a hollow cylinder of length L and inner radius a and outer radius b, as shown in Figure 6.4.4. The material has resistivity ρ.

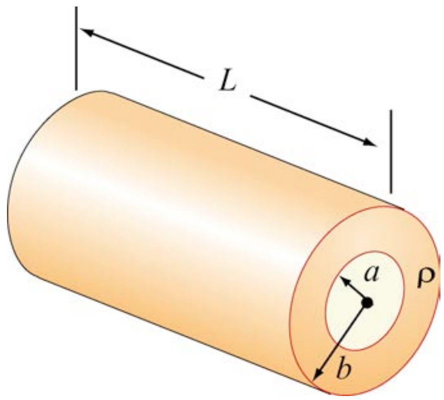

Figure 6.4.4 A hollow cylinder.

(a) Suppose a potential difference is applied between the ends of the cylinder and produces a current flowing parallel to the axis. What is the resistance measured?

(b) If instead the potential difference is applied between the inner and outer surfaces so that current flows radially outward, what is the resistance measured?

Solution:

(a) When a potential difference is applied between the ends of the cylinder, the flow of charge is parallel to the axis. In this case, the cross-sectional area is $A = \pi(b^2 - a^2)$, and the resistance is given by

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi(b^2 - a^2)}.$$

(b) Consider a differential element, which is made up of a thin cylinder of inner radius r and outer radius $r + dr$ and length L. Its contribution to the resistance of the system is given by

$$dR = \frac{\rho\, dl}{A} = \frac{\rho\, dr}{2\pi r L},$$

where $A = 2\pi r L$ is the area normal to the direction of current. The total resistance of the system becomes

$$R = \int_a^b \frac{\rho\, dr}{2\pi r L} = \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right).$$

6.5 Conceptual Questions

1. Two wires A and B of circular cross-section are made of the same metal and have equal lengths, but the resistance of wire A is four times greater than that of wire B. Find the ratio of their cross-sectional areas.

2. From the point of view of atomic theory, explain why the resistance of a material increases as its temperature increases.

6.6 Additional Problems

6.6.1 Current and Current Density

A sphere of radius 10 mm that carries a charge of $8\text{ nC} = 8\times10^{-9}\text{C}$ is whirled in a circle at the end of an insulated string. The angular frequency is $100\pi\text{ s}^{-1}$.

(a) What is the basic definition of current in terms of charge?

(b) What average current does this rotating charge represent?

(c) What is the average current density over the area traversed by the sphere?

6.6.2 Resistance of a Cone

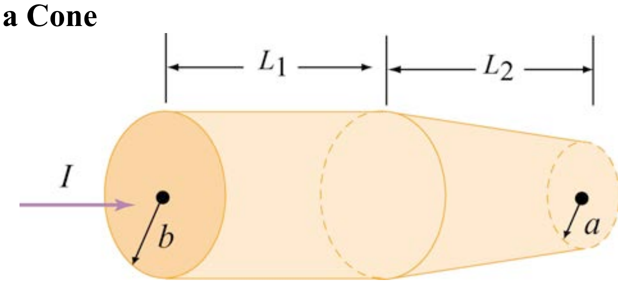

Figure 6.6.1

A copper resistor of resistivity ρ is in the shape of a cylinder of radius *b* and length L_1 appended to a truncated right circular cone of length L_2 and end radii *b* and *a* as shown in Figure 6.6.1.

(a) What is the resistance of the cylindrical portion of the resistor?

(b) What is the resistance of the entire resistor? (Hint: For the tapered portion, it is necessary to write down the incremental resistance *dR* of a small slice, *dx*, of the resistor at an arbitrary position, *x*, and then to sum the slices by integration. If the taper is small, one may assume that the current density is uniform across any cross section.)

(c) Show that your answer reduces to the expected expression if $a = b$.

(d) If $L_1 = 100\text{ mm}$, $L_2 = 50\text{ mm}$, $a = 0.5\text{ mm}$, and $b = 1.0\text{ mm}$, what is the resistance?

6.6.3 Current Density and Drift Speed

(a) A group of charges, each with charge *q*, moves with velocity $\vec{\mathbf{v}}$. The number of particles per unit volume is *n*. What is the current density $\vec{\mathbf{J}}$ of these charges, in magnitude and direction? Make sure that your answer has units of $\text{A}\cdot\text{m}^{-2}$.

(b) We want to calculate how long it takes an electron to get from a car battery to the starter motor after the ignition switch is turned. Assume that the current flowing is 115 A, and that the electrons travel through copper wire with cross-sectional area 31.2 mm^2 and length 85.5 cm. What is the current density in the wire? The number density of the conduction electrons in copper is $8.49\times10^{28}\text{ /m}^3$. Given this number density and the current density, what is the drift speed of the electrons? How long does it take for an electron starting at the battery to reach the starter motor? [Ans.: $3.69\times10^6\text{ A/m}^2$, $2.71\times10^{-4}\text{ m/s}$, 52.5 min.]

6.6.4 Current Sheet

A *current sheet*, as the name implies, is a plane containing currents flowing in one direction in that plane. One way to construct a sheet of current is by running many parallel wires in a plane, say the yz-plane, as shown in Figure 6.6.2(a). Each of these wires carries current *I* out of the page, in the $-\hat{\mathbf{j}}$ direction, with *n* wires per unit length in the *z*-direction, as shown in Figure 6.6.2(a). Then the current per unit length in the *z* direction is nI. We will use the symbol *K* to signify current per unit length, so that $K = nl$ here.

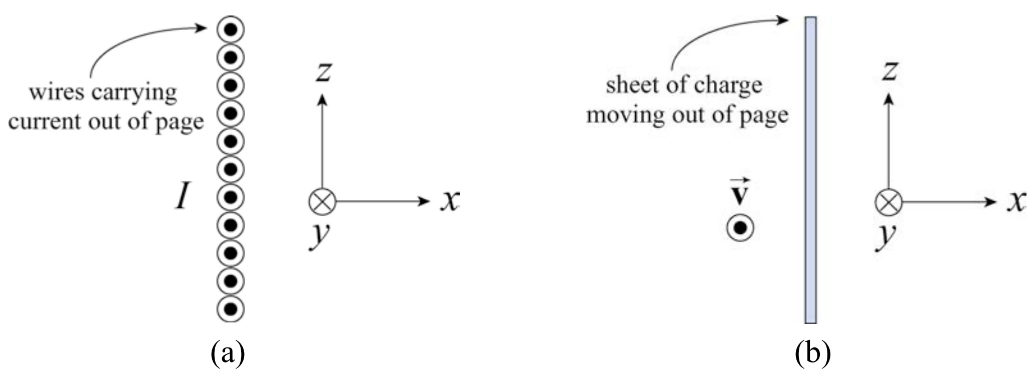

Figure 6.6.2 A current sheet.

Another way to construct a current sheet is to take a non-conducting sheet of charge with fixed charge per unit area σ and move it with some speed in the direction you want current to flow. For example, in Figure 6.6.2(b), we have a sheet of charge moving out of the page with speed v. The direction of current flow is out of the page.

(a) Show that the magnitude of the current per unit length in the *z* direction, K, is given by σv. Check that this quantity has the proper dimensions of current per length. This is in fact a vector relation, $\vec{\mathbf{K}}(t) = \sigma \vec{\mathbf{v}}(t)$, since the sense of the current flow is in the same direction as the velocity of the positive charges.

(b) A belt transferring charge to the high-potential inner shell of a Van de Graaff accelerator at the rate of 2.83 mC/s. If the width of the belt carrying the charge is 50 cm and the belt travels at a speed of 30 m/s, what is the surface charge density on the belt? [Ans.: 189 μC/m^2]

6.6.5 Resistance and Resistivity

A wire with a resistance of 6.0 Ω is drawn out through a die so that its new length is three times its original length. Find the resistance of the longer wire, assuming that the resistivity and density of the material are not changed during the drawing process. [Ans.: 54 Ω].

6.6.6 Charge Accumulation at the Interface

Figure 6.6.3 shows a three-layer sandwich made of two resistive materials with resistivities ρ_1 and ρ_2. From left to right, we have a layer of material with resistivity ρ_1 of width $d/3$, followed by a layer of material with resistivity ρ_2, also of width $d/3$, followed by another layer of the first material with resistivity ρ_1, again of width $d/3$.

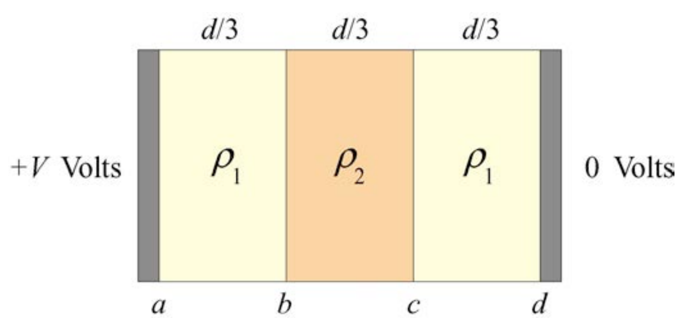

Figure 6.6.3 Charge accumulation at interface.

The cross-sectional area of all of these materials is *A*. The resistive sandwich is bounded on either side by metallic conductors (black regions). Using a battery (not shown), we maintain a potential difference *V* across the entire sandwich, between the metallic conductors. The left side of the sandwich is at the higher potential (*i.e.*, the electric fields point from left to right).

There are four interfaces between the various materials and the conductors, which we label *a* through *d*, as indicated on the sketch. A steady current *I* is directed in the sandwich from left to right, corresponding to a current density $J = I / A$.

(a) What are the electric fields $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ in the two different dielectric materials? To obtain these fields, assume that the current density is the same in every layer. Why must this be true? [Ans.: All fields point to the right, $E_1 = \rho_1 I / A$, $E_2 = \rho_2 I / A$; the current densities must be the same in a steady state, otherwise there would be a continuous buildup of charge at the interfaces to unlimited values.]

(b) What is the total resistance *R* of this sandwich? Show that your expression reduces to the expected result if $\rho_1 = \rho_2 = \rho$. [Ans.: $R = d\left(2\rho_1 + \rho_2\right) / 3A$; if $\rho_1 = \rho_2 = \rho$, then $R = d\rho / A$, as expected.]

(c) As we move from right to left, what are the changes in potential across the three layers, in terms of *V* and the resistivities? [Ans.: $V\rho_1 / (2\rho_1 + \rho_2)$, $V\rho_2 / (2\rho_1 + \rho_2)$, $V\rho_1 / (2\rho_1 + \rho_2)$, summing to a total potential drop of *V*, as required].

(d) What are the charges per unit area, σ_a through σ_d, at the interfaces? Use Gauss's Law and assume that the electric field in the conducting caps is zero.
[Ans.: $\sigma_a = -\sigma_d = 3\varepsilon_0 V\rho_1 / d\left(2\rho_1 + \rho_2\right)$, $\sigma_b = -\sigma_c = 3\varepsilon_0 V\left(\rho_2 - \rho_1\right) / d\left(2\rho_1 + \rho_2\right)$.]

(e) Consider the limit $\rho_2 >> \rho_1$. What do your answers above reduce to in this limit?